Software Reliability Modeling Based on Zero-truncated and/or Zero-inflated Compound Distributions

Jingchi Wu, Tadashi Dohi*, and Hiroyuki Okamura
1-4-1 Kagamiyama, Higashi-hiroshima, Hiroshima, Japan
d220580@hiroshima-u.ac.jp, dohi@hiroshima-u.ac.jp, okamu@hiroshima-u.ac.jp
*corresponding author

Abstract—Software reliability growth models (SRGMs) are used to assess quantitative software reliability and to monitor/control software testing progress. During almost the last five decades, SRGMs based on non-homogeneous Poisson processes (NHPPs) have gained much popularity for describing the stochastic behavior of the cumulative number of software faults detected in testing phase, because of their tractability and goodness-of-fit performances. Grottke and Trivedi (2005) proposed an interesting NHPP-based modeling framework, called all-stage zero-truncated NHPP-based SRGMs, and showed their goodness-of-fit and predictive performances with several actual software development project data. In this paper we further generalize their idea on all-stage zero-truncation by introducing zero-truncated and/or zero-inflated compound probability distributions. Throughout comprehensive numerical experiments, we compare our generalized modeling frameworks with the existing ones in terms of goodness-of-fit and predictive performances, and show that the zero-truncated NHPP-based SRGMs are still attractive more than the others including the non-truncated SRGMs.
Keywords–Software reliability, Software reliability growth models, NHPP, zero-truncated compound distributions, zero-inflated compound distributions, goodness-of-fit, prediction.

I. Introduction

In software development processes described by waterfall development model, software reliability growth models (SRGMs) are used to assess quantitative software reliability and to monitor/control software testing [21], [25]. Since the quantitative software reliability is defined as the probability that software failures caused by faults do not occur for a given period of time, the probabilistic behavior of software fault-detection process in testing phase is modeled by any stochastic counting process to estimate the software reliability. During almost the last five decades, the SRGMs based on non-homogeneous Poisson processes (NHPPs) have gained much popularity for describing the stochastic behavior of the cumulative number of detected software faults, because of their tractability and goodness-of-fit performance. Especially, almost all NHPP-based SRGMs are characterized by the bounded mean value functions, which are often called the *finite failure models*. Goel-Okumoto SRGM [9], Goel SRGM [10], Ohba SRGM [26], Yamada et al.'s SRGM [35], Zhao and Xie SRGM [36] are the representative finite failure NHPP-based SRGMs. The SRGMs mentioned above are corresponding to the typical fault-detection time distributions such as exponential distribution, Weibull distribution, truncated logistic distribution, and gamma distributions with/without specified scale parameter.

While these finite failure NHPP-based SRGMs are well motivated by the software debugging mechanism from a population (software program) with unknown number of inherent faults, they possess an uncommon property that the inter-failure time distributions are all *defective* [16], [31], *i.e.*, the defective probability distributions with non-zero mass part at infinity exist. This property does not enable us to assess some useful software reliability measures such as mean time to failure (MTTF) and mean time between failures (MTBF), because the resulting finite moments of the inter-failure time distributions always diverge. Hishitani et al. [15] proposed an intuitive approximate method to assess MTBF with the degenerate probability distributions of the inter-failure times. Although their idea was to use simply the normalized distribution functions without non-zero mass part at infinity, such an approach is not convinced theoretically and does not essentially lead to the accurate assessment of MTTF and MTBF as software reliability measures.

Hence the recent research trends in NHPP-based SRGM are to find out more appropriate fault-detection time distributions including Pareto distribution [1], lognormal distribution [2], [29], log-logistic distribution [11], Burr-type distributions [18], extreme-type distributions [27], truncated normal distribution [29], *etc.*, and to investigate the *infinite failure models*. Musa and Okumoto [24] proposed the logarithmic Poisson execution time model, called Musa and Okumoto SRGM, and examined an applicability of the infinite failure model to software reliability [25]. Littlewood [20] and Cretois and Gaudoin [5] applied another infinite failure model, called the power-law model or the Duane model [8], to describe the software fault-detection phenomena. Very recently, Li et al. [19] treated eleven infinite-failure NHPP-based SRGMs and compared them with their associated finite-failure NHPP-based SRGMs. In this paper we focus on a different modeling technique from the common finite-failure NHPP-based SRGMs. Grottke and

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Trivedi [12], [13] considered an approach to deal with the defective inter-failure time distributions in the finite failure NHPP-based SRGMs, and developed the so-called all-stage truncated NHPP-based SRGMs with an intermediate feature between the finite failure models with bounded mean value functions and the infinite failure models with unbounded mean value functions. It should be noted that their novel modeling approach seems to be quite interesting, because the zero-truncation of the Poisson distribution is used for the fault-detection time distribution, and the resulting all-stage truncated models are reduced to simple NHPP-based SRGMs. We further extend/generalize their all-stage truncated NHPP-based SRGMs by introducing the zero-inflated Poisson and binomial distributions. The zero-inflated as well as zero-truncated counting processes are observed in various fields in natural calamities [3], dental epidemiology [4], reliability engineering [17], ultrasound localization microscopy [6], horticulture [14], biological control [32]. So, the objective of this paper is to provide a rich modeling framework with zero-truncated and/or zero-inflated compound distributions in software reliability modeling.

The paper is organized as follows. In Section 2, we summarize the existing NHPP-based SRGMs with/without truncation, where we mainly introduce the results in Grottke and Trivedi [12], [13]. In Section 3, we introduce the zero-truncated/zero-inflated Poisson and binomial distributions. By combining six non-trivial Poisson and binomial distributions, we develop three novel NHPP-based SRGMs with the bounded mean value function, in addition to the common NHPP-based SRGMs without truncation/inflation and the all-stage truncated NHPP-based SRGMs with truncated Poisson distribution. Section 4 is devoted to numerical experiments for comparison of our NHPP-based SRGMs in terms of goodness-of fit and predictive performances. Finally, the paper is concluded with some remarks in Section 5.

II. Software Reliability Modeling

1. NHPP-based SRGMs

In NHPP-based SRGMs, the cumulative number of software faults detected by time t (≥ 0), $\{X(t), t \geq 0\}$, obeys the Poisson probability mass function (p.m.f.) with parameter $\Lambda(t)$;

$$\Pr\{X(t) = x\} = \frac{\{\Lambda(t)\}^x e^{-\Lambda(t)}}{x!}, \tag{1}$$

where $\Lambda(t) = \mathrm{E}[X(t)]$ is the mean value function and denotes the expected cumulative number of software faults detected up to time t. More specifically, suppose that the number of inherent software faults before testing, say, at time $t = 0$, is given by a non-negative integer-value N, and that each software fault in the program is detected at independent and identically distributed random testing time having the continuous non-degenerate cumulative distribution function (c.d.f.) $G(t)$ with probability density function (p.d.f.) $g(t)$. Then, the conditional p.m.f. of the cumulative number of software faults detected by time t is given by the binomial distribution;

$$\begin{aligned}\Pr\{X(t) = x|N\} &= B(x; N, G(t)) \\ &= \binom{N}{x} G(t)^x \overline{G}(t)^{N-x}, \end{aligned} \tag{2}$$

where $\overline{G}(\cdot) = 1 - G(\cdot)$ and $B(x; N, p)$ is the binomial p.m.f. with parameters N (integer) and p $(0 < p < 1)$. Since the number of inherent software faults, N, is still unknown even after completing the system testing, it is appropriate to make the assumption that N is also regarded as an integer-valued random variable. If N obeys the Poisson distribution with finite mean ν (> 0), then we have the following unconditional p.m.f.;

$$\begin{aligned}\Pr\{X(t) = x\} &= \sum_{n=0}^{\infty} \Pr\{X(t) = x|N\} \Pr\{N = n\} \\ &= \frac{\{\nu G(t)\}^x e^{-\nu G(t)}}{x!} \end{aligned} \tag{3}$$

with $\Pr\{N = n\} = P(n; \nu) = \nu^n \exp(-\nu)/n!$. Hence, under the plausible assumptions above, it is straightforward to see that $X(t)$ follows an NHPP with mean value function $\Lambda(t) = \nu G(t)$ $(0 < \nu < \infty)$.

The NHPP-based SRGMs with mean value function in Eq.(3) is classified into the finite failure model with bounded mean value functions, $\lim_{t\to\infty} \Lambda(t) = \nu < \infty$. Since $X(t)$ is an NHPP, it can be also regarded as a continuous-time non-homogeneous Markov chain [31]. Let $r_{i-1}(t)$ be the transition rate of an NHPP from state $i-1$ to state i $(= 1, 2, \ldots)$. Then the transition rates do not depend on the state i and are identically given by $r(t) = \nu g(t)$. Figure 1 illustrates the transition diagram of an NHPP with transition rate $r(t) = \nu g(t)$ for arbitrary time t. Table I presents the representative fault-detection time distributions $G(t)$. Okamura and Dohi [30] developed SRATS, software reliability assessment tool on spreadsheet, and implemented the maximum likelihood estimation algorithms based on the EM (Expectation-Maximization) principle with the fault-detection time distributions in Table I.

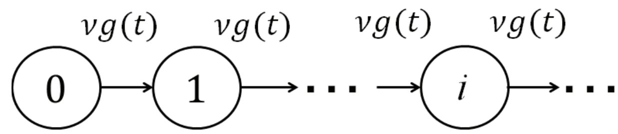


Figure 1: Transition diagram of NHPP-based SRGM without truncation.

2. All-stage Zero-truncated NHPP-based SRGMs

Grottke and Trivedi [12], [13] assumed that the software program involves at least one software fault before the software testing, *i.e.*, the fault-free probability is zero before the testing, and that the number of inherent software faults before the testing, N, obeys the zero-truncated Poisson distribution. The zero-truncated Poisson (ZTP) distribution is a special discrete probability distribution, which is a variant of the Poisson distribution, but it excludes zero mass. The main difference

TABLE I: Representative fault-detection time distributions.

c.d.f.	$G(t)$ $(b, c{>}0)$
Exp [9]	$G(t) = 1 - e^{-bt}$
Gamma [35], [36]	$G(t) = \int_0^t \frac{c^b s^{b-1} e^{-cs}}{\Gamma(b)} ds$
Pareto [1]	$G(t) = 1 - \left(\frac{c}{t+c}\right)^b$
TruncNormal [29]	$G(t) = \frac{F(t)-F(0)}{1-F(0)}$, $F(t) = \frac{1}{\sqrt{2\pi}b} \int_t^{-\infty} e^{-\frac{(s-c)^2}{2b^2}} ds$
LogNormal [2], [29]	$G(t) = \frac{1}{\sqrt{2\pi}b} \int_{\log(t)}^{-\infty} e^{-\frac{(s-c)^2}{2b^2}} ds$
TruncLogist [26]	$G(t) = \frac{F(t)-F(0)}{1-F(0)}$, $F(t) = \frac{1}{1+e^{-\frac{t-c}{b}}}$
LogLogist [11]	$G(t) = \frac{1}{1+e^{-\frac{\log(t)-c}{b}}}$
TruncEVMax [27]	$G(t) = \frac{F(t)-F(0)}{1-F(0)}$, $F(t) = e^{-e^{-\frac{t-c}{b}}}$
LogEVMax [27]	$G(t) = e^{-e^{-\frac{\log(t)-c}{b}}}$

between the ZTP distribution and the Poisson distribution is that the Poisson distribution is defined on all non-negative integers, including zero, while the ZTP distribution is only defined on positive integers. This means that if a random variable follows a ZTP distribution, it will never take a zero value. For example, in the field of software reliability, we might encounter a situation where the number of software faults that occur within a certain period of time is at least 1. In this case, we cannot use the Poisson distribution to model the number of software faults, because the Poisson distribution allows zero faults to occur. Instead, we should use the ZTP distribution because it only includes positive integer values. Consider the transition behavior in a continuous-time non-homogeneous Markov chain from state 0 to state 1. Let $R(t|0, X(0) = 0)$ be the software reliability as the probability that no software fault is detected for $(0, t]$ with $X(0) = 0$, where the observation point is $t = 0$. Then, we have

$$R(x|0, X(0) = 0) = \sum_{n=1}^{\infty} \overline{G}(x)^n \cdot \frac{\nu^n}{n!} \frac{e^{-\nu}}{1 - e^{-\nu}} = \frac{e^{\nu \overline{G}(x)} - 1}{e^{\nu} - 1}. \tag{4}$$

For the transition rate $r_{i-1}(t)$ $(i = 1, 2, \ldots)$, a specific transition rate $r_0(t)$ from state 0 to state 1 for the above Markov chain is given by

$$r_0(t) = \frac{-dR(t|0, X(0) = 0)/dt}{R(t|0, X(0) = 0)} = \frac{\nu g(t)}{1 - e^{-\nu \overline{G}(t)}}. \tag{5}$$

Though this model, called the *one-stage zero-truncated process*, is truncated at $N = 0$ for the number of inherent software faults before the testing, the remaining transition rates from state $i - 1$ to state i at time $t = t_{i-1}$ $(i = 2, 3, \ldots)$ are different from $r_0(t)$. For the residual number of software

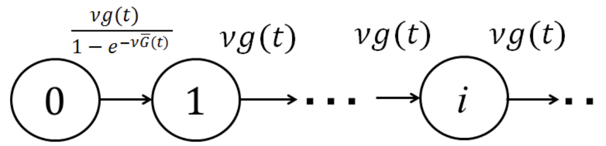


Figure 2: Transition diagram of one-stage zero-truncated SRGM.

faults, $M(t) = N - X(t)$, the software reliability with $i \geq 2$ is given by [12], [13];

$$\begin{aligned} & R(x|t_{i-1}, X(t_{i-1}) = i - 1) \\ & = \Pr\{M(t_{i-1} + x) - M(t_{i-1}) = i - 1 | M(t_{i-1}) = i - 1\} \\ & = e^{-\{\Lambda(t_{i-1}+x) - \Lambda(t_{i-1})\}}. \end{aligned} \tag{6}$$

Hence the transition rates $r_{i-1}(t)$ at time t_{i-1} $(i = 2, 3, \ldots)$ are reduced to

$$r_{i-1}(t) = \frac{-dR(t|t_{i-1}, X(t_{i-1}) = i - 1)/dt}{R(t|t_{i-1}, X(t_{i-1}) = i - 1)} = \nu g(t), \tag{7}$$

which are exactly the same transition rates as the common NHPP without truncation. Figure 2 is the transition diagram of the one-stage truncated process at origin. We note that this stochastic counting process with one-stage truncation is no longer a Poisson process and is not tractable for the analysis. The fundamental idea on all-stage zero-truncated NHPP by Grottke and Trivedi [12], [13] is to replace all transition rates by $r_0(t)$ in Eq.(5). Replacement of all the defective transition probabilities for two successive events by the non-defective ones enables us to develop another NHPP-based SRGM with an arbitrary $G(t)$. More precisely, it can be derived that

$$\begin{aligned} \Pr\{N = n | X(t) = i - 1\} &= \frac{[\nu \overline{G}(t)]^{n-(i-1)}}{(n - (i - 1))!} \frac{e^{-\nu \overline{G}(t)}}{1 - e^{-\nu \overline{G}(t)}} \\ &= \frac{[\nu \overline{G}(t)]^{n-(i-1)}}{(n - (i - 1))!} \frac{1}{e^{\nu \overline{G}(t)} - 1} \end{aligned} \tag{8}$$

for $n \geq i$. Hence, the software reliability function is given by

$$\begin{aligned} R(x|t_{i-1}, X(t_{i-1}) = i - 1) &= \sum_{n=i}^{\infty} \left(\frac{\overline{G}(t_{i-1} + x)}{\overline{G}(t_{i-1})} \right)^{n-(i-1)} \\ &\times \frac{[\nu \overline{G}(t_{i-1})]^{n-(i-1)}}{(n - (i - 1))!} \frac{1}{e^{\nu \overline{G}(t_{i-1})} - 1} \\ &= \frac{e^{\nu \overline{G}(t_{i-1}+x)} - 1}{e^{\nu \overline{G}(t_{i-1})} - 1} \end{aligned} \tag{9}$$

for $i \geq 1$. Then we derive the transition rate for all-stage zero-truncated NHPP-based SRGM as

$$r_{i-1}(t) = \frac{\nu g(t)}{1 - e^{-\nu \overline{G}(t)}} \tag{10}$$

for $i \geq 1$. It is easily shown that this model is reduced to an NHPP-based SRGM with unbounded mean value function, i.e., $\lim_{t \to \infty} \Lambda(t) \to \infty$. Figure 3 depicts the transition diagram of all-stage zero-truncated NHPP-based SRGM. It is also shown

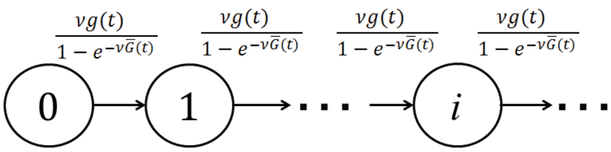


Figure 3: Transition diagram of all-stage zero-truncated NHPP-based SRGM.

in [12], [13] that all-stage truncated NHPP-based SRGMs have attractive features for both NHPPs with/without truncation.

III. Generalizations

1. Zero-truncated and/or Zero-inflated Compound Distributions

Next, our concern is the derivation of alternative compound distributions with the zero-truncated (ZT) and/or the zero-inflated (ZI) distributions for the binomial distribution $B(m;n,p)$ and the Poisson distribution $P(m;\nu)$ [7], [22]. The zero-truncated binomial (ZTB) and zero-truncated Poisson (ZTP) distributions are defined by the binomial and Poisson distributions with positive support $m = 1, 2, 3, \cdots$, having the p.m.f.s; $\Pr\{X = m|X \geq 1\}$ and $\Pr\{N = m|N \geq 1\}$, respectively. Grottke and Trivedi [12], [13] just focused on only the zero-truncated Poisson distribution in Eq.(4). In addition, we introduce the zero-inflated binomial (ZIB) and zero-inflated Poisson (ZIP) distributions whose p.m.f.s have the special mass parts at $N = 0$ and $X = 0$, respectively. The ZIB/ZIP distribution is a special discrete probability distribution used to describe the phenomenon of observing too many zeros in certain situations. The ZIB/ZIP distribution consists of two parts: deterministic zeros and a part that follows a binomial/Poisson distribution. The ZIB/ZIP distribution assumes two types of zeros, one is "structural zeros", i.e., zeros due to some inherent structure or mechanism, and the other is "sampling zeros", i.e., zeros randomly sampled from a Poisson process. In the field of software reliability, we may encounter situations where no fault is found in a series of software tests. This is because these tests did not trigger any fault, which are "structural zeros", or although these tests triggered faults, they were not detected for various reasons such as randomness, incompleteness of testing, etc., which are "sampling zeros". In this case, we can use the ZIB/ZIP distribution to model the software testing results more accurately. For instance, the ZIB distribution, $ZIB(m;p,\omega)$, is given by

$$\Pr\{X = m\} = \begin{cases} \omega + (1-\omega)q^n, & m = 0 \\ (1-\omega)B(m;n,p), & m \geq 1 \end{cases} \tag{11}$$

with $q = 1-p$ and the zero-inflation parameter $\omega \in [-q^n/(1-q^n), 1]$. When $\omega = -q^n/(1-q^n)$, the ZIB distribution is reduced to the ZTB distribution;

$$\Pr\{X = m\} = \frac{\binom{n}{m} p^m q^{n-m}}{1-q^n}, \quad m \geq 1. \tag{12}$$

In this sense, since the ZIB distribution is a generalization of the ZTB distribution, we deal with only the ZIB distribution hereafter. For the Poisson case, we can derive the ZIP and ZTP distributions, $ZIP(m;\nu,\omega)$ and $ZTP(m;\nu)$, as well. Table II presents the common binomial/Poisson distributions, ZTB/ZTP distributions and ZIB/ZIP distributions, where ω_1 $(-1/(e^\nu - 1) \leq \omega_1 \leq 1)$ and ω_2 $(-q^n/(1-q^n) \leq \omega_2 \leq 1)$ denote the respective inflation parameters, ω_1 and ω_2, for ZIP and ZIB distributions, respectively.

Based on the results in Table II, we derive the compound distributions. The compound distributions of the binomial and Poisson distributions (B-P) and the binomial and ZTP distributions (B-ZTP) are trivial. Here we derive the compound distributions with combinations of the binomial distribution and ZIP (B-ZIP), ZIB and the Poisson distribution (ZIB-P), ZIB and ZTP (ZIB-ZTP), ZIB and ZIP (ZIB-ZIP). For all the combinations above, we obtain the zero count probability as

$$\Pr\{X = 0\} = \Pr\{N = 0\} + \sum_{n=1}^{\infty} \Pr\{X = 0|N = n\} \times \Pr\{N = n\}. \tag{13}$$

For example, in the combination of the binomial distribution and the ZTP distribution in [12], [13], we have

$$\Pr\{X = m\} = \sum_{n=m}^{\infty} \binom{n}{m} p^m q^{n-m} \cdot \frac{\nu^n}{n!} \frac{e^{-\nu}}{1-e^{-\nu}} = \frac{(\nu p)^m}{m!} \frac{e^{-\nu p}}{1-e^{-\nu}} \tag{14}$$

for $m \geq 1$, where

$$\Pr\{X = 0\} = \sum_{n=1}^{\infty} q^n \cdot \frac{\nu^n}{n!} \frac{e^{-\nu}}{1-e^{-\nu}} = \frac{e^{-\nu p} - e^{-\nu}}{1-e^{-\nu}}. \tag{15}$$

In Table III, we summarize six compound distributions, where $ZTP(m;\nu)$, $ZIP(m;\nu,\omega_1)$ and $ZIB(n;p,\omega_2)$ are the ZTP with parameter ν, the ZIP with parameters (ν, ω_1) and the ZIB with parameter (p, ω_2), respectively.

2. New Class of Finite Failure NHPP-based SRGMs

Based on the results in Table III, we develop the associated all-stage truncated and/or all-stage inflated NHPP-based SRGMs. For the brevity, we just give a combination of the common binomial distribution and the ZIP distribution. Since the software reliability is given by

$$R(t|0, X(0) = 0) = \omega_1 + (1-\omega_1)e^{-\nu G(t)}, \tag{16}$$

the transition rate $r_0(t)$ can be obtained as

$$r_0(t) = \frac{(1-\omega_1)\nu g(t) e^{-\nu G(t)}}{\omega_1 + (1-\omega_1)e^{-\nu G(t)}}. \tag{17}$$

Hence, the mean value function of the corresponding all-stage inflated NHPP-based SRGM (B-ZIP) is derived as

$$\Lambda(t) = -\ln\left(R(t|0, X(0) = 0)\right) = -\ln\left(\omega_1 + (1-\omega_1)e^{-\nu G(t)}\right). \tag{18}$$

TABLE II: Zero-truncated and zero-inflated distributions.

p.m.f.	Common p.m.f.	ZT p.m.f.	ZI p.m.f.
$P(m;\nu)$ (Poisson)	$\frac{\nu^m}{m!}e^{-\nu}(m \geq 0)$	$\frac{\nu^m}{m!(e^{\nu}-1)}\ (m \geq 1)$	$\omega_1 + (1-\omega_1)e^{-\nu}(m=0)$ $(1-\omega_1)\frac{\nu^m}{m!}e^{-\nu}(m \geq 1)$
$B(m;n,p)$ (Binomial)	$\binom{n}{m}p^m q^{n-m}(m \geq 0)$	$\frac{\binom{n}{m}p^m q^{n-m}}{1-q^n}(m \geq 1)$	$\omega_2 + (1-\omega_2)q^n(m=0)$ $(1-\omega_2)\binom{n}{m}p^m q^{n-m}(m \geq 1)$

TABLE III: Compound distributions.

p.m.f.	$B(n,p)$	$ZIB(n,p,\omega_2)$
$P(m;\nu)$	$e^{-\nu p}\frac{(\nu p)^m}{m!}(m \geq 0)$	$\omega_2 + (1-\omega_2)e^{-\nu p}$ $(m=0)$ $(1-\omega_2)e^{-\nu p}\frac{(\nu p)^m}{m!}$ $(m \geq 1)$
$ZTP(m;\nu)$	$\frac{e^{-\nu p}-e^{-\nu}}{1-e^{-\nu}}\ (m=0)$ $\frac{e^{-\nu p}}{1-e^{-\nu}}\frac{(\nu p)^m}{m!}\ (m \geq 1)$	$\omega_2 + (1-\omega_2)\frac{e^{-\nu p}-e^{-\nu}}{1-e^{-\nu}}$ $(m=0)$ $(1-\omega_2)\frac{e^{-\nu p}}{1-e^{-\nu}}\frac{(\nu p)^m}{m!}$ $(m \geq 1)$
$ZIP(m;\nu,\omega_1)$	$\omega_1 + (1-\omega_1)e^{-\nu p}$ $(m=0)$ $(1-\omega_1)e^{-\nu p}\frac{(\nu p)^m}{m!}$ $(m \geq 1)$	$\omega_1+\omega_2-\omega_1\omega_2$ $+(1-\omega_1)(1-\omega_2)e^{-\nu p}$ $(m=0)$ $(1-\omega_1)(1-\omega_2)e^{-\nu p}\frac{(\nu p)^m}{m!}$ $(m \geq 1)$

TABLE IV: Mean value functions for all-stage truncated and all-stage inflated NHPP-based SRGMs.

$\Lambda(t)$	$B(n,G(t))$	$ZIB(n,G(t),\omega_2)$
$P(m;\nu)$	$\nu G(t)$	$\ln\left(\frac{1}{\omega_2+(1-\omega_2)e^{-\nu G(t)}}\right)$
$ZTP(m;\nu)$	$\ln\left(\frac{1-e^{-\nu}}{e^{-\nu G(t)}-e^{-\nu}}\right)$	$\ln\left(\frac{1-e^{-\nu}}{\omega_2(1-e^{-\nu})+(1-\omega_2)(e^{-\nu G(t)}-e^{-\nu})}\right)$
$ZIP(m;\nu,\omega_1)$	$\ln\left(\frac{1}{\omega_1+(1-\omega_1)e^{-\nu G(t)}}\right)$	$\ln\left(\frac{1}{\omega_1+\omega_2-\omega_1\omega_2+(1-\omega_1)(1-\omega_2)e^{-\nu G(t)}}\right)$

Table IV presents the mean value functions for all-stage truncated and all-stage inflated NHPP-based SRGMs. From these results, by replacing p by $G(t)$ in Table II, it can be seen that the NHPP with unbounded mean value function is only B-ZTP (a combination of the common binomial distribution and the ZTP distribution) by Grottke and Trivedi [12], [13]. In other words, it is pointed out in Table IV that B-ZIP, ZIB-P, ZIB-ZTP and ZIB-ZIP give the bounded mean value functions, where B-ZIP is exactly same as ZIB-P.

Note that SRGMs with zero-inflation include additional parameters ω_1 and/or ω_2. These parameters imply the inflation probability if $0 \leq \omega_i \leq 1$ $(i = 1, 2)$, but should be regarded as model parameters to be estimated in the range of $-1/(e^{\nu} - 1) \leq \omega_i \leq 1$, because the all-stage truncated NHPP-based SRGM in [12], [13] is a special case in our ZIB-ZTP with $\omega_2 = 0$. This fact indicates that the likelihood function based on our ZIB-ZTP is not less than that based on B-ZTP in the reference [12], [13]. The special interest is that the modeling framework developed here is distribution-free, so substituting an arbitrary c.d.f. into $G(t)$ leads to various reliability growth patterns in unbounded mean value function.

IV. Numerical Experiments

1. Data Sets

In the experiments, we compare all the NHPP-based SRGMs; existing NHPP (B-P), B-ZTP, B-ZIP, ZIB-ZTP and ZIB-ZIP with the general fault-detection time distributions $G(t)$. For the comparative purpose, we assume nine distribution functions in

TABLE V: Data Sets

Data set	Testing weeks	No. faults	Source	Nature of system
DS1	17	54	SYS2 [23]	Real time command and control system
DS2	14	38	SYS3 [23]	Real time command and control system
DS3	19	120	Release2 [34]	Tandem software system
DS4	12	61	Release3 [34]	Tandem software system
DS5	14	9	NASA-supported project [33]	Inertial navigating system
DS6	20	66	DS1 [28]	Embedded application for printer
DS7	33	58	DS2 [28]	Embedded application for printer
DS8	30	52	DS3 [28]	Embedded application for printer

TABLE VI: Comparison of goodness-of-fit performances.

	Best Zero-Truncation (B-ZTP)	Best B-ZIP	Best ZIB-ZTP	Best ZIB-ZIP	Best Existing NHPP (B-P)
DS1	65.3604439 (TruncNormal)	75.0527041 (LogLogist)	67.3604439 (TruncNormal)	77.8964183 (TruncEVMax)	73.0527000 (LogLogist)
DS2	56.1390325 (TruncEVMax)	63.6936843 (LogEVMax)	57.2423972 (LogNormal)	65.4254006 (LogEVMax)	61.6937300 (LogEVMax)
DS3	87.2307592 (TruncNormal)	89.2571816 (TruncNormal)	89.2307592 (TruncNormal)	95.5477433 (Exp)	87.2571900 (TruncNormal)
DS4	51.1235411 (TruncLogist)	53.0515099 (TruncLogist)	53.1235411 (TruncLogist)	55.1060030 (TruncLogist)	51.0515100 (TruncLogist)
DS5	29.9102340 (Exp)	31.6817298 (Exp)	31.6817298 (Exp)	33.6811795 (Exp)	29.9105000 (Exp)
DS6	99.4530997 (TruncNormal)	110.8305884 (LogEVMax)	94.1210768 (Gamma)	106.4052658 (LogEVMax)	108.8308000 (LogEVMax)
DS7	127.1340002 (TruncLogist)	128.4896745 (TruncNormal)	129.1340002 (TruncLogist)	128.0907893 (TruncLogist)	126.4897000 (TruncNormal)
DS8	110.1312924 (TruncLogist)	119.4701575 (LogLogist)	112.1312924 (TruncLogist)	124.7401497 (LogLogist)	117.4702000 (LogLogist)

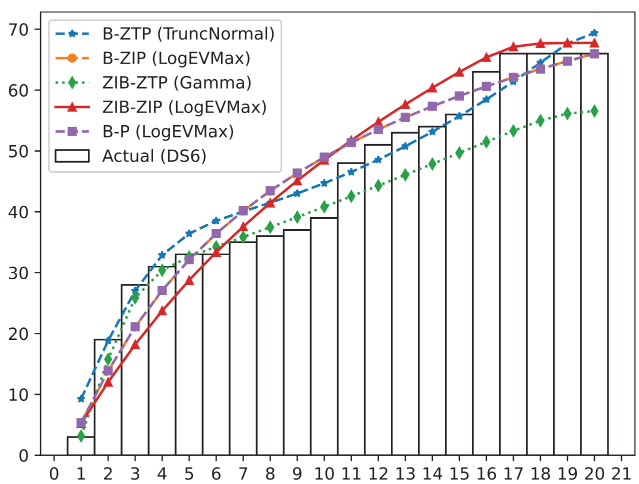


Figure 4: Estimation behaviors of the cumulative number of software faults in DS6.

Table I; Exp [9], Gamma [35], [36], Pareto [1], TruncNormal [29], LogNormal [2], [29], TruncLogist [26], TruncEVMax [27], LogEVMax [27]. In Table V, we present eight data sets to use for the analysis, where all software development project data sets are software fault-detection time interval data (group data) observed in actual software development projects.

2. Goodness-of-Fit Performance

First of all, we compare all the NHPP-based SRGMs (existing (B-P), zero-truncation (B-ZTP), B-ZIP, ZIB-ZTP, ZIB-ZIP) with nine kinds of fault-detection time c.d.f.s in terms of goodness-of-fit performance. The model parameters are estimated by means of the maximum likelihood method, where the best model is selected based on the smallest Akaike information criterion (AIC):

$$\mathrm{AIC} = -2\mathrm{MLL} + 2\pi, \quad (19)$$

with the maximized log-likelihood, MLL, and the degree of freedom (number of free parameters), π.

In Figure 4, we show the behaviors of the mean value functions for B-P, B-ZTP, B-ZIP, ZIB-ZTP and ZIB-ZIP with DS6 in Table V, where the model in brackets denotes the best fault-detection time distribution in Table I. It is observed that B-P shows a different trend from ZIB-ZIP, and that both B-ZTP and/or ZIB-ZTP somewhat underestimate the cumulative number of software faults more than B-P and ZIB-ZIP in DS6. Looking at the behaviors of the mean value function, the difference among B-ZTP, B-ZIP and ZIB-ZTP is not so remarkable. To check more detailed differences, Table VI presents the comparison of goodness-of-fit performances based on the AIC. From the results with eight data sets, it is seen that all-stage truncated NHPP-based SRGMs [12], [13] (B-ZTP) provides the best goodness-of-fit in five data sets (DS1, DS2, DS3, DS5, DS8), and that the common NHPP-based SRGM (P-B) does in two data sets (DS4, DS7). Since ZIB-ZTP gives the best result for only DS6, at first look, our proposed SRGMs (B-ZIP, ZIB-ZTP, ZIB-ZIP) might be considered as poor fitting performances. It is worth noting however that the differences on AIC between ZIB-ZTP and the best SRGM in all data sets are less than 2. Because ZIB-ZTP contains an additional parameter ω_1 comparing with B-P and B-ZTP, the goodness-of-fit performances for ZIB-ZTP and the best SRGM in DS1-DS5 and DS7-DS8 are almost similar. Instead, in DS6, our ZIB-ZTP shows much better performance than the other four SRGMs. The observation above implies that our compound distribution model (ZIB-ZTP) has advantages to the common NHPP-based SRGM (B-P) and the all-stage truncated NHPP-based SRGMs (B-ZTP), and outperforms the other SRGM modeling frameworks in all data sets.

3. Predictive Performance

Next we investigate the predictive performances on our NHPP-based SRGMs for the unknown pattern of the cumulative number of software faults in the future. As the training data, we use the software fault counts observed at 20%, 50% and 80% points of the entire data, and predict the cumulative number of faults in the remaining time periods corresponding to 80%, 50% and 20% data. The predictive mean squared error (PMSE) is used:

$$\mathrm{PMSE} = \frac{\sqrt{\sum_{i=n+1}^{n+l}(\Lambda(t_i) - x_i)^2}}{l}, \tag{20}$$

where (t_i, x_i) $(i = n+1, \ldots, n+l)$ are the group data for validation and l $(= 1, 2, \ldots)$ are the prediction lengths. In Figure. 5, we plot the prediction behaviors of the cumulative number of software faults at 20%, 50% and 80% observation points, respectively, with DS6. In the early (20%) and middle (50%) testing phases, the common NHPP-based SRGMs (B-P) tend to rather under-predict the number of faults detected in the future. More specifically, it is observed that B-ZTP and ZIB-ZIP make the over-predictions in the early testing, and that all the SRGMs tend to under-predict the fault counts. In the late (80%) testing phase (see Figure. 4 (c)), we find that only ZIB-ZIP gives a stable prediction.

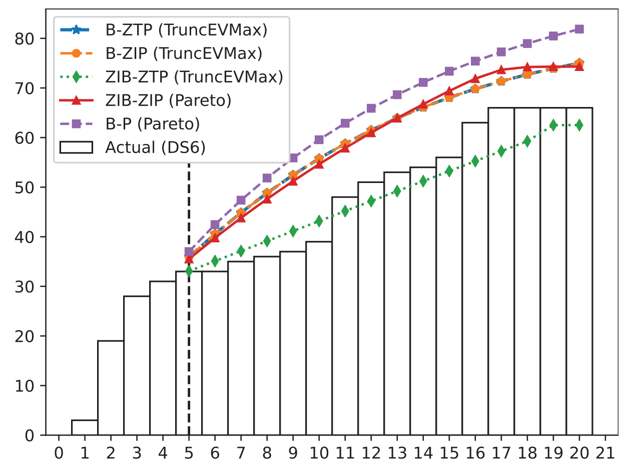
To examine the predictive performances more accurately, we compare the minimum PMSEs for all the NHPP-based SRGMs at 20%, 50% and 80% observation points in Tables VII, VIII and IX, respectively. It can be seen that B-ZTP gives the best predictive performances in four data sets, three data sets and four data sets out of eight in Tables VII, VIII and IX, respectively. The point to be addressed here is that our new SRGMs (B-ZIP, ZIB-ZTP, ZIB-ZIP) could make the best predictions in three, four and three data sets at 20%, 50% and 80% observation points, respectively. Instead, the predictive performances on the common NHPP-based SRGM (B-P) are relatively poor compared with the zero-truncated and/or zero-inflated NHPP-based SRGMs. These results suggest us that the zero-truncated and/or zero-inflated NHPP-based SRGMs have great potential to assess quantitative software reliability more accurately.

However, note that the predictive performances in Tables VII, VIII and IX are based on the best prediction models with the minimum PMSEs in each model category. In other words, it is not feasible to know the best prediction model in advance at each observation point. In Figure. 6, we show the behaviors of the predictions by respective SRGMs with the minimum AICs at each observation point when DS6 is analyzed. It can be seen that the common NHPP-based SRGM (B-P) tends to predict a smaller cumulative number of software faults than the other SRGMs in both early and middle testing phases. On one hand, we see that ZIB-ZIP makes a plat prediction in the late testing phase. Tables X, XI and XII present the comparison of PMSEs with the minimum AIC at 20%, 50% and 80% observation points, respectively. So, we use the best goodness-of-fit models for the prediction for the remaining testing periods. In the early testing phase at 20% observation point, it is found that B-ZTP gives the smaller PMSEs in four data sets (DS3, DS4, DS6, DS7). In the middle testing phase at 50% observation point, we can check ZIB-ZIP could provide the best prediction results in four cases. In the late testing phase at 80% observation point, B-ZTP won in four cases, and our new NHPP-based SRGMs result cloud show the best predictions in three data sets. In the above feasible predictions, we found that the all-stage truncated NHPP-based SRGM in [12], [13] (B-ZTP) could give nice predictive performances on average, although it does not always the really best prediction model.

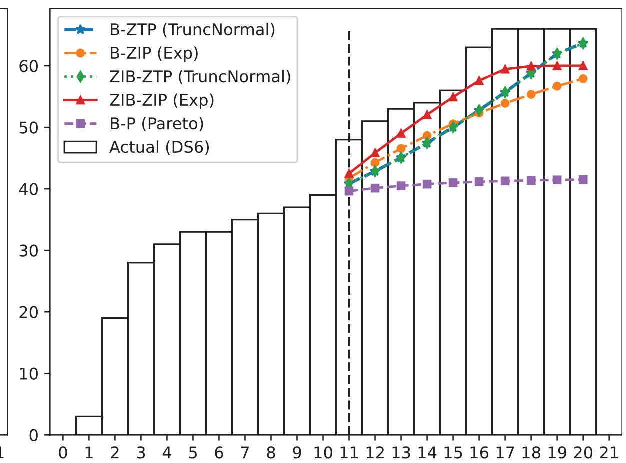
It should be emphasized that we do not aim at finding out the best prediction model in this paper, because both the goodness-of-fit and predictive performances strongly depend on the kind of software fault count data. Our claim in this paper is that the zero-truncated and/or zero-inflated NHPP-based SRGMs including the all-stage truncated NHPP-based SRGM in [12], [13], B-ZTP, B-ZIP, ZIB-ZTP and ZIB-ZIP, are useful to quantify the software reliability, and can be regarded as competing SRGMs for the common NHPP-based SRGM (B-P). This fact tells us that the naive treatment of the zero count in software reliability modeling affects the accurate goodness-of-fit and predictive performances on the software fault counts.

V. Conclusion

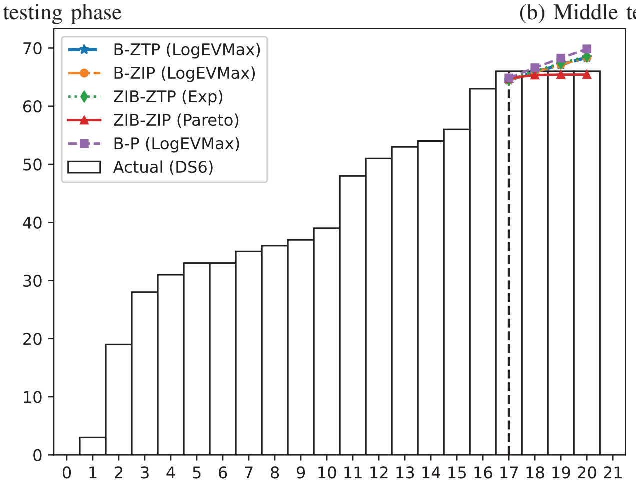
In this paper, we have proposed a unified modeling framework on zero-truncated and/or zero-inflated NHPP-based SRGMs, which are generalizations of the existing all-stage truncated NHPP-based SRGMs in [12], [13]. We have developed three novel NHPP-based SRGMs with bounded mean value functions, by introducing the zero-inflated binomial and Poisson distributions. In numerical experiments, we have compared five NHPP-based SRGM frameworks with nine baseline probability distributions (fault-detection time distributions)



(a) Early testing phase



(b) Middle testing phase



(c) Late testing phase

Figure 5: Prediction behaviors of the cumulative number of software faults with the minimum PMSE in DS6.

TABLE VII: Comparison of predictive performances at 20% point.

20% Best PMSE					
	Zero-Truncation (B-ZTP)	B-ZIP	ZIB-ZTP	ZIB-ZIP	Existing NHPP (B-P)
DS1	3.95295 (Gamma)	3.95297 (LogLogist)	2.30117 (TruncLogist)	1.09084 (TruncLogist)	1.291475 (TruncLogist)
DS2	0.587803274 (TruncEVMax)	8.150215975 (LogEVMax)	0.509822998 (TruncEVMax)	1.7011249 (TruncLogist)	2.10945 (TruncEVMax)
DS3	7.187367945 (Gamma)	7.187250064 (Gamma)	14.7753837 (LogEVMax)	7.3814763 (Gamma)	7.756989 (Gamma)
DS4	5.26728768 (TruncNormal)	12.27198957 (Exp)	12.27198957 (Exp)	12.280689 (Exp)	11.02741 (Pareto)
DS5	0.46577 (Pareto)	0.46772 (Exp)	0.46772 (Exp)	0.8406568 (Pareto)	0.52506 (Exp)
DS6	2.634281604 (TruncEVMax)	2.946731141 (TruncEVMax)	4.570474898 (TruncEVMax)	2.7320466 (Pareto)	3.688794 (Pareto)
DS7	2.053909619 (TruncNormal)	6.830437556 (Pareto)	6.830437556 (Pareto)	6.8304955 (Pareto)	4.122789 (Exp)
DS8	6.958507505 (Exp)	6.961564841 (LogEVMax)	6.952764921 (LogEVMax)	6.9584784 (Pareto)	4.721361 (TruncNormal)

TABLE VIII: Comparison of predictive performances at 50% point.

50% Best PMSE					
	Zero-Truncation (B-ZTP)	B-ZIP	ZIB-ZTP	ZIB-ZIP	Existing NHPP (B-P)
DS1	0.84767 (TruncNormal)	0.99188 (TruncNormal)	3.35876 (LogNormal)	0.99205 (TruncNormal)	0.990064 (TruncNormal)
DS2	0.4458199 (TruncEVMax)	2.4065462 (Exp)	0.6131751 (TruncEVMax)	2.4065544 (Exp)	2.399617 (Pareto)
DS3	9.37152 (LogEVMax)	9.0648332 (Pareto)	8.1871851 (Exp)	2.0169678 (Exp)	5.305319 (LogEVMax)
DS4	4.1029494 (Pareto)	4.1056317 (Pareto)	6.9741587 (Exp)	2.7624573 (TruncLogist)	3.077433 (Pareto)
DS5	0.13579 (Gamma)	0.15121 (TruncEVMax)	0.13579 (Gamma)	0.1512251 (TruncEVMax)	0.117158 (Pareto)
DS6	1.2897265 (TruncNormal)	6.0509329 (Exp)	3.2231267 (TruncNormal)	6.0509313 (Exp)	6.034253 (Pareto)
DS7	1.1467313 (LogEVMax)	1.1467315 (LogEVMax)	3.6902654 (TruncLogist)	0.3586782 (TruncEVMax)	1.187932 (Gamma)
DS8	3.455164 (TruncLogist)	2.986172 (TruncLogist)	1.3476605 (LogLogist)	0.7465237 (Exp)	2.996425 (TruncLogist)

TABLE IX: Comparison of predictive performances at 80% point.

80% Best PMSE					
	Zero-Truncation (B-ZTP)	B-ZIP	ZIB-ZTP	ZIB-ZIP	Existing NHPP (B-P)
DS1	0.66412 (LogNormal)	0.66425 (LogNormal)	1.37705 (TruncLogist)	0.66425 (LogNormal)	0.66412345 (LogNormal)
DS2	0.39318661 (Exp)	0.3931864 (Exp)	1.9148542 (Exp)	0.3931864 (Exp)	0.39274963 (TruncLogist)
DS3	0.27094611 (TruncNormal)	0.2879378 (TruncNormal)	2.8394542 (Exp)	2.8122989 (TruncNormal)	0.28547051 (TruncNormal)
DS4	0.97778658 (TruncNormal)	0.7855031 (TruncNormal)	1.7950549 (Exp)	1.7728895 (TruncLogist)	0.78745896 (TruncNormal)
DS5	0.38178 (Exp)	0.72071 (LogNormal)	0.80770 (LogEVMax)	0.809102 (LogEVMax)	0.38661832 (Exp)
DS6	1.17935222 (LogEVMax)	1.1793522 (LogEVMax)	1.5 (Exp)	0.7064471 (Pareto)	1.16337724 (LogEVMax)
DS7	2.21485012 (TruncLogist)	2.2148353 (TruncLogist)	0.914732 (Exp)	0.4662558 (Exp)	2.22908663 (TruncLogist)
DS8	0.26835297 (Gamma)	0.3793148 (LogEVMax)	1.0274023 (Exp)	0.7830395 (LogNormal)	0.37824431 (LogEVMax)

with eight actual software fault count data sets in terms of goodness-of-fit and predictive performances. As the numerical results, we have found that the goodness-of-fit performances between the all-stage truncated NHPP-based SRGM in [12], [13] and our all-stage binomial inflated and Poisson truncated NHPP-based SRGMs (ZIB-ZTP) were almost similar, and the former tended to outperform the other SRGMs on the predictive performances in many cases. However, it is worth mentioning that the best NHPP-based SRGMs should be carefully checked in a more generalized modeling framework, because the model selection affects the prediction accuracy of quantitative software reliability, and the existing ones (B-P, B-ZTP) do not always provide satisfactory prediction performances. For example, in Table 6 with DS3, DS4, DS7, and DS8, the prediction performance of ZIB-ZIP is higher than B-P and B-ZTP. In that sense, the generalization framework of NHPP-based SRGMs proposed in this paper will be useful to explore more appropriate NHPP-based SRGMs with given baseline probability distributions.

In the future, we will analyze the software fault-detection time-domain data, since we treated only the group data on software fault counts in this paper. Also, we will compare our all-stage zero-truncated and/or zero-inflated NHPP-based SRGMs (B-ZIP, ZIB-ZTP, ZIB-ZIP) having the bounded mean value functions with the other infinite-failure NHPP-based SRGMs [19] in a comprehensive numerical study.

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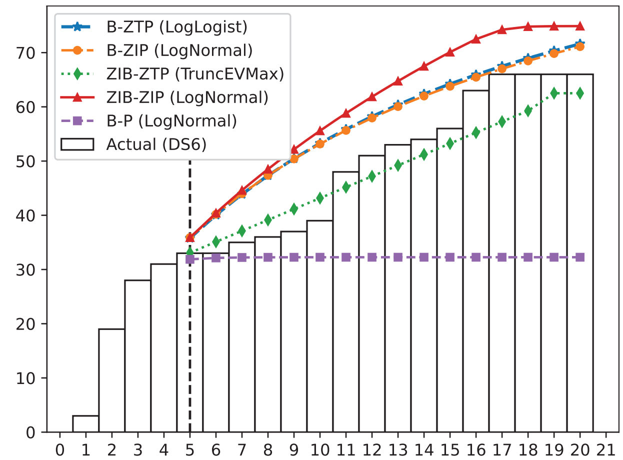
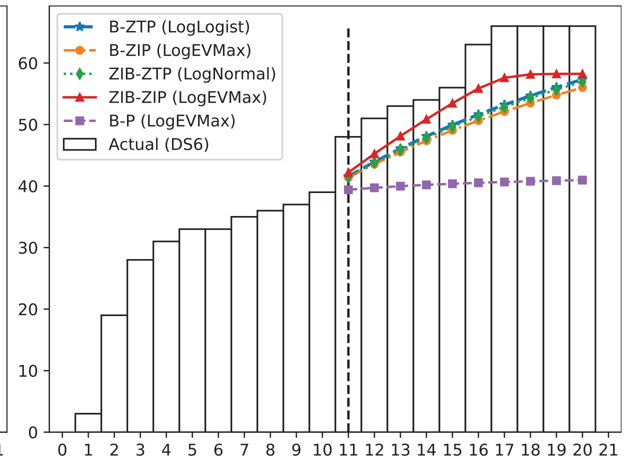
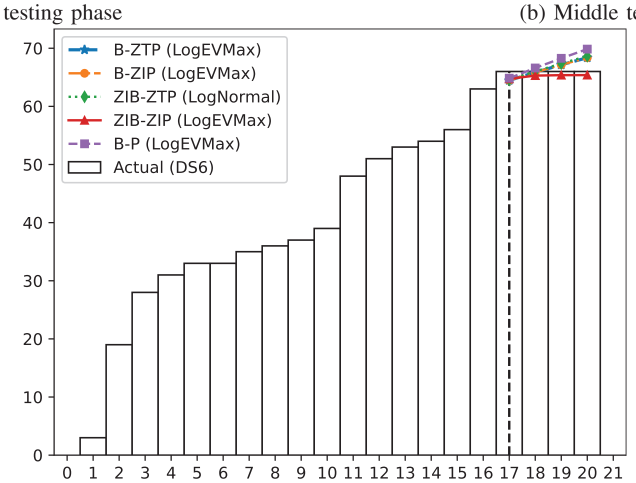


(a) Early testing phase



(b) Middle testing phase



(c) Late testing phase

Figure 6: Prediction behaviors of the cumulative number of software faults with the minimum AIC in DS6.

TABLE X: Comparison of predictive performances with the minimum AIC at 20% point.

20% Best AIC					
	Zero-Truncation (B-ZTP)	B-ZIP	ZIB-ZTP	ZIB-ZIP	Existing NHPP (B-P)
DS1	6.6855494 (Exp)	6.6855645 (Exp)	6.6855645 (Exp)	6.6855297 (Exp)	6.8967679 (Exp)
DS2	11.1306797 (LogNormal)	47.4796588 (Gamma)	4.7166643 (TruncNormal)	4.2142664 (Gamma)	2.2933280 (Gamma)
DS3	13.2157774 (Exp)	13.3881754 (Exp)	14.7792338 (Exp)	13.3881783 (Exp)	13.3973626 (Exp)
DS4	10.4343286 (Exp)	12.2719896 (Exp)	12.2719896 (Exp)	12.2806893 (Exp)	11.4725403 (Exp)
DS5	0.4677291 (Exp)	0.4677162 (Exp)	0.4677162 (Exp)	1.4116732 (Exp)	0.5250604 (Exp)
DS6	4.5265151 (LogLogist)	5.4682638 (LogNormal)	4.5704749 (TruncEVMax)	5.4682614 (LogNormal)	5.4683146 (LogNormal)
DS7	6.1831982 (LogEVMax)	6.8439394 (TruncNormal)	6.8439394 (TruncLogist)	6.8439394 (TruncLogist)	6.8434442 (TruncLogist)
DS8	14.5499547 (LogLogist)	6.9615648 (LogEVMax)	6.9527649 (LogEVMax)	7.3898640 (TruncNormal)	7.4655386 (TruncLogist)

TABLE XI: Comparison of predictive performances with the minimum AIC at 50% point.

50% Best AIC					
	Zero-Truncation (B-ZTP)	B-ZIP	ZIB-ZTP	ZIB-ZIP	Existing NHPP (B-P)
DS1	0.8706052 (TruncLogist)	1.5965533 (TruncLogist)	4.0612593 (TruncLogist)	1.5965536 (TruncLogist)	1.5902013 (TruncLogist)
DS2	2.1419990 (LogLogist)	3.0095595 (LogEVMax)	2.2022811 (LogEVMax)	3.0095584 (LogEVMax)	3.0100633 (LogEVMax)
DS3	10.2491593 (Exp)	10.2491576 (Exp)	8.1871851 (Exp)	2.0169678 (Exp)	9.3387968 (Exp)
DS4	26.9405431 (TruncLogist)	26.9404918 (TruncLogist)	6.9741587 (TruncLogist)	2.7624573 (TruncLogist)	23.1740533 (TruncLogist)
DS5	0.2226098 (Exp)	0.2226121 (Exp)	0.2226121 (Exp)	0.2222182 (Exp)	0.1855971 (Exp)
DS6	6.0167511 (LogLogist)	6.2014415 (LogEVMax)	6.6535705 (LogNormal)	6.2014431 (LogEVMax)	6.2016228 (LogEVMax)
DS7	1.1467313 (LogEVMax)	1.1467315 (LogEVMax)	3.6958621 (LogEVMax)	1.0118397 (LogEVMax)	1.2677341 (LogNormal)
DS8	3.4551640 (TruncLogist)	2.9861720 (TruncLogist)	1.8402898 (TruncLogist)	1.8402895 (TruncEVMax)	2.9964254 (TruncLogist)

TABLE XII: Comparison of predictive performances with the minimum AIC at 80% point.

80% Best AIC					
	Zero-Truncation (B-ZTP)	B-ZIP	ZIB-ZTP	ZIB-ZIP	Existing NHPP (B-P)
DS1	0.7478621 (TruncNormal)	1.7575206 (TruncLogist)	1.3770526 (TruncLogist)	1.7575323 (TruncLogist)	1.7576970 (TruncLogist)
DS2	1.2115371 (TruncEVMax)	0.5561438 (LogEVMax)	1.9148542 (LogNormal)	0.5561440 (LogEVMax)	0.5601448 (LogEVMax)
DS3	0.2709461 (TruncNormal)	0.2879378 (TruncNormal)	2.8394542 (Exp)	2.8299185 (Exp)	0.2854705 (TruncNormal)
DS4	1.0860148 (TruncLogist)	1.0437989 (TruncLogist)	1.7950549 (TruncLogist)	1.7728895 (TruncLogist)	1.0479680 (TruncLogist)
DS5	0.3817806 (Exp)	0.8592032 (Exp)	0.8592032 (Exp)	0.8593403 (Exp)	0.3866183 (Exp)
DS6	1.1793522 (LogEVMax)	1.1793522 (LogEVMax)	1.5000000 (LogNormal)	1.1793523 (LogEVMax)	1.1633772 (LogEVMax)
DS7	2.2148501 (TruncLogist)	2.2148353 (TruncLogist)	0.9147320 (LogLogist)	0.8292214 (LogEVMax)	2.2290866 (TruncLogist)
DS8	0.3111114 (TruncLogist)	0.9978557 (TruncLogist)	1.0274023 (Pareto)	0.9978559 (TruncLogist)	0.9978573 (TruncLogist)

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