Performance Degradation Assessment of Rolling Bearing Based on Difference of Eigenvalues in Random Matrix Theory

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Abstract—The difference of eigenvalues index based on Random Matrix Theory (RMT) was proposed to overcome the problem that traditional feature extraction methods hardly extract effective feature when the sample data is large and severe noise in signal, which further affects the accuracy of bearing state detection. Firstly, a technique based on matrix randomization is proposed, which can construct a large dimensional random matrix that meeting the requirement of random matrix theory through matrix randomization processing. Secondly, the convergence characteristics of eigenvalues and the difference between matrix eigenvalues in Marchenko-Pastur (M-P) law based on random matrix theory can achieve noise reduction, and a rolling bearing eigenvalue difference index is proposed to reduce noise interference. Finally, rolling bearing full life data collected by Intelligent Maintenance System (IMS) was used for application research. The experimental results show that: the feasibility and effectiveness of M-P law in the field of bearing state monitoring can be verified, and the proposed index not only can detect the abnormal occurrence in advance, but also can accurately describe the degradation process of rolling bearing.

Keywords: *Random Matrix Theory(RMT); Marchenko-Pastur (M-P) law; Rolling bearing; Performance degradation assessment*

1. INTRODUCTION

As key component of rotating machinery, the running quality of rolling bearings determines the overall stability and efficient safety of equipment operation [1]. Therefore, how to construct a degradation index that can effectively reflect the operating status of rolling bearing has become one of the research hotspots in field of equipment condition monitoring. In the research of bearing performance degradation assessment, most scholars usually extract signal features from bearing vibration signals and construct degradation index to characterize the process of bearing degradation. Traditional indexes use time-domain, frequency-domain, and time-frequency domain features of bearing vibration signal for analysis in terms of feature extraction and degradation index construction of rolling bearing [2]. Kumar et al. used Kullback-Leibler (K-L) divergence as an index of bearing health degradation [3]. Tong et al. decomposed the vibration acceleration signal using Variational Mode Decomposition (VMD), and extracted the singular values and relative energy characteristics of each decomposed signal, which can effectively assess the severity of bearing degradation [4]. Permutation Entropy (PE) was utilized by Zhang et al. as bearing degradation feature, the Entropy Energy Rate (EER) was improved to construct a performance degradation index for rolling bearing, which is more sensitive to early failures and more consistent with the development trend of bearing failures [5]. Cheng et al. introduced the concept of grey system theory based on nonlinear dynamics of entropy and proposed a Grey Entropy (GreyEn) index. Compared with other entropy indexes, GreyEn can effectively evaluate the vibration performance degradation of rolling bearing in complex situations [6]. These above methods are suitable for monitoring situations with small data, but there are problems such as low efficiency and large computational complexity when the scale of sample datasets is large.

Random Matrix Theory (RMT), as an important mathematical tool for statistical analysis [7]. At present, it has two main branches, such as: single ring theory and M-P law, which have been successfully applied in big data processing fields [8]. Yang et al. proposed a method for evaluating operation status of power grids based on random matrix theory and qualitative trend analysis, which can not only detect whether the current state of the power grid is stable, but also whether there is a negative operating trend in the current power grid [9]. Hu investigated an anomaly node detection method for Wireless Sensor Networks (WSN) based on random matrix theory, which solved the problems of low recall accuracy and long detection time in traditional detection methods in the field of wireless transmission [10]. Based on random matrix theory, the Electroencephalogram (EEG) signals from different channels were analyzed by Sarma et al., emotion recognition is achieved by constructing different correlation matrices and performing feature decomposition on the matrices, which improves the classification accuracy of emotion recognition technology [11]. Based on excellent data processing characteristics of random matrix theory in other fields and its advantages in state monitoring, Ni et al. applied the single ring theory into the field of bearing state monitoring, a bearing performance degradation assessment method based on random matrix theory was proposed for the first time. The constructed index can detect abnormal occurrences of bearings as early as possible. But they do not accurately describe the degradation process of bearings at each stage [12]. Given the good application effect of random matrix theory, this theory was applied in the field of rolling bearing condition monitoring in this paper. Random matrix theory is utilized to mine the distribution characteristics of bearing data, and the performance degradation index of the difference

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between the maximum and minimum eigenvalue is built to assess the bearing performance degradation.

In summary, the M-P law in random matrix theory is applied to the field of rolling bearing state monitoring, and constructs a difference of eigenvalues index to assessment the full life degradation process of rolling bearings in this paper. The application results show the effectiveness and feasibility of this method.

2. INTRODUCTION TO M-P LAW IN RMT

Random matrix theory, as an important mathematical tool for statistical analysis, which is continuously studied by scholars on its statistical laws and application value. Marchenko and Pastur proposed their limit spectral distribution M-P law base on RMT in 1967 [13], and this law has been applied in various fields recently.

For a matrix $X \in C^{M \times N}$ with the scale is $M \times N$, if matrix X meets the following three conditions:

(1) If the mean of its internal elements is 0 and the variance σ^2 is 1 in matrix X.

(2) The elements meet the requirements of independent and identically distributed distribution in matrix X.

(3) As $c = M/N(M \to \infty, N \to \infty)$, and the ratio of c does not change.

Therefore, the probability density distribution of its eigenvalues follows the M-P law after constructing the covariance matrix for matrix X and performing eigenvalue decomposition. The empirical spectrum distribution of eigenvalues reads [14]:

$$f_{M-P}(\lambda) = \begin{cases} \dfrac{1}{2\pi\lambda c\sigma^2}\sqrt{(b-\lambda)(\lambda-a)}, & a \le |\lambda| \le b \\ 0 & , \text{ else} \end{cases} \tag{1}$$

Where, a and b denote the lower and upper limit of the empirical spectrum, respectively, and $a = \sigma^2(1-\sqrt{c})^2$, $b = \sigma^2(1+\sqrt{c})^2$。

Meanwhile, when the number of rows and columns satisfies the condition that $\lim\limits_{N\to\infty}\dfrac{M}{N} = c$ $(0<c \le 1)$ remains constant, the limit expression of the eigenvalues of the Wishart random matrix is [15]:

$$\begin{cases} \lim\limits_{N\to\infty}\lambda_{\min} = \dfrac{\sigma^2}{N}a = \dfrac{\sigma^2}{N}(\sqrt{N}-\sqrt{M})^2 \\ \lim\limits_{N\to\infty}\lambda_{\max} = \dfrac{\sigma^2}{N}b = \dfrac{\sigma^2}{N}(\sqrt{N}+\sqrt{M})^2 \end{cases} \tag{2}$$

Where, σ^2 is the variance of matrix X; M and N denote the number of rows and columns of matrix X, respectively; a and b are the convergence value of the minimum and maximum eigenvalues in the M-P law, respectively.

3. CONSTRUCTION OF PERFORMANCE DEGRADATION INDICATORS BASED ON M-P LAW

3.1. Randomization of Rolling Bearing Data

Setting the sampling frequency of signal acquisition card as P, and conducting Q times of sampling within the sampling time T. Thus, all monitored data can form a column vector $l(t_i)$ at the sampling time $t_i(t_i \in T)$. Construct data source matrix $L \in C^{P\times Q}$ containing all bearing life information by collecting time series $l(t_i)$ at different moment.

$$L = \left[l(t_1), l(t_2), \cdots, l(t_i), \cdots\right](i = 1,2,3,\cdots,Q) \tag{3}$$

In order to meet the requirement of large sample of high-dimensional random matrix, similar matrix $Z(t_i) \in C^{P\times 1}$ is constructed to expand the data collected, and P rows of column vector of bearing data collected at t_i moment are divided into k ($k \ge 2$)segments, and k submatrices $\tilde{H}_i(t_i)(i = 1,2,3\cdots,k)$ can be formed. The i -th matrix $\tilde{Z}(t_i)$ in the similar matrix $Z(t_i)$ can be formed through (4).

$$\tilde{Z}(t_i)_i = \sum_{j=1}^{i}\theta_j\tilde{H}_i(t_i)/i(i = 1,2,3,\cdots,k) \tag{4}$$

Where, θ_j $(\theta_j \in [0.95, 1.05])$ is random number generated within a certain range, which is used for matrix randomization construction.

The constructed simulation matrix Z is

$$Z(t_i) = \left[\tilde{Z}_1(t_i); \tilde{Z}_2(t_i); \tilde{Z}_3(t_i); \cdots; \tilde{Z}_k(t_i)\right](k \ge 2) \tag{5}$$

Repeating the above operations O times to generate N similar matrices $Z_o(t_i)(o = 1,2,3,\cdots,N)$, and the large scale monitoring matrix constructed at t_i moment will be obtained.

$$\tilde{X}(t_i) = \left[l(t_i), Z_1(t_i), Z_2(t_i), \cdots, Z_O(t_i)\right] \tag{6}$$

When the ratio of matrix rows and columns c should be in $(0,1]$, the distribution of its eigenvalues follows a certain law, as shown in (1). Thus, the scale of matrix must be reconstructed, and finally large scales random matrix $X(t_i) \in C^{M\times N}$ is constructed for eigenvalue decomposition and extraction at t_i moment.

Constructing a high-dimensional monitoring matrix $X(t_i)$, and calculate its covariance matrix from (7):

$$B_n(t_i) = \frac{\sigma^2}{N}X_i(t_i)X_i(t_i)^T \tag{7}$$

Where, σ^2 is the variance of matrix $X(t_i)$.

After eigenvalue decomposition of matrix $B_n(t_i)$, the maximum eigenvalue $\lambda_{\max}$ and minimum eigenvalue $\lambda_{\min}$ can be extracted respectively.

Considering that the collected bearing signals contain two components: (1) Real vibration signals that can reflect the operating status of rolling bearing; (2) Noise signals that exist due to external environment or collection system interference, and these noise signals are relatively stable, but can interfere with the determination of the bearing operating states. The bearing vibration signal can be decomposed, and the internal components of the extracted eigenvalues are shown in (8):

$$\begin{cases} \lambda_{\min} = \lambda_{v-\min} + \lambda_{n} \\ \lambda_{\max} = \lambda_{v-\max} + \lambda_{n} \end{cases} \tag{8}$$

Where, $\lambda_{v-\min}$ and $\lambda_{v-\max}$ are the minimum and maximum covariance matrices of actual vibration signal, respectively; λ_{n} is the eigenvalues of the unknown noise covariance matrix.
To eliminate noise interference and improve the robustness of detection index, a degradation index D is constructed using the difference between the maximum and minimum eigenvalues:

$$D = \lambda_{\max} - \lambda_{\min} = \lambda_{v-\max} - \lambda_{v-\min} \tag{9}$$

The final constructed index D can eliminate the interference caused by stable noise in the system, and this article applies it to assess the bearing performance.
In summary, the process of the degradation assessment algorithm in this article is shown in Figure 1.

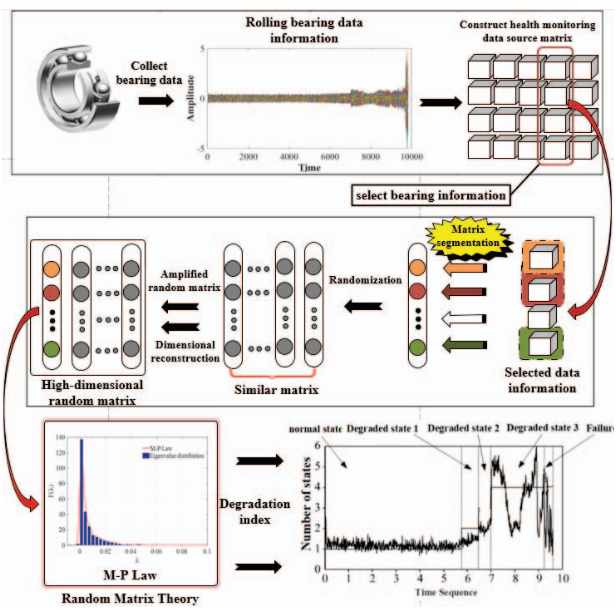


Figure 1. The flowchart of bearing performance assessment algorithm

Step 1: Using sensors to collect vibration signal, and the data collected at each time is regarded as columns of matrix, Splicing the bearing data collected at different times, and finally a full life signal matrix of the bearing collected at different times could be constructed.
Step 2: Selecting the vibration data of rolling bearing at a certain time for segmentation, randomization, and the similar matrix was created to expand the number of elements in matrix. Thus, a large dimensional random matrix with the ratio of rows to columns $c \in (0,1]$ was finally constructed through matrix changes.
Step 3: The distribution of eigenvalues in bearing covariance matrix is decomposed and extracted based on M-P law, we extracted the maximum and minimum eigenvalues to build a degradation index D , and finally used it for assessing performance degradation of rolling bearing.

4. APPLICATION RESEARCH

4.1. Data source and processing

The rolling bearing datasets were collected by Intelligent Maintenance System (IMS) center. IMS bearing full life experiment vibration signal acquisition platform as shown in Figure 2. In this experiment, two acceleration sensors were respectively arranged in the horizontal and vertical directions of rolling bearing to collect acceleration vibration signal, the sampling frequency of sensors is 20kHz, and the sampling interval is set to 10 minutes. It was found that the outer ring of the bearing 1 is damaged during operation when the experiment stops, this process lasted for 164 hours, resulting in 984 sets of collected data files. We conduct research on bearing 1 and analyze the vibration data collected by its vertical direction sensor in this article.

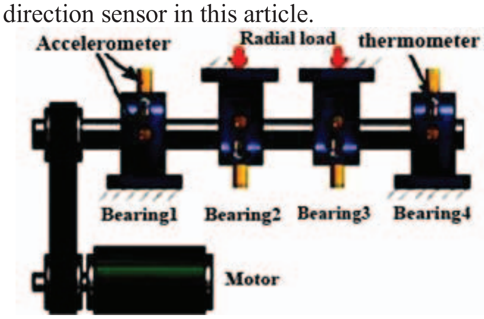


Figure 2. IMS bearing full life experiment vibration signal acquisition platform

The original vibration signal of IMS rolling bearing 1 is shown in Figure 3. It can be found that the amplitude of bearing vibration increases with time, and the more serious the bearing fault is, the more intense the amplitude changes. The original data collected has certain reference value for bearing status recognition and performance evaluation. However, due to the complexity of datasets, it is hardly to accurately reflect the abnormal occurrence point and the real degradation process of bearing 1. Thus, it is necessary to mine this data and construct better degradation index to describe the degradation process of bearing. According to the content described in this article, we analyze and process IMS bearing dataset.

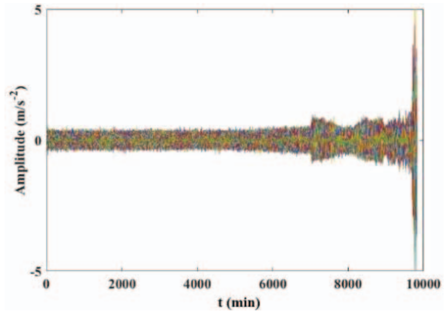
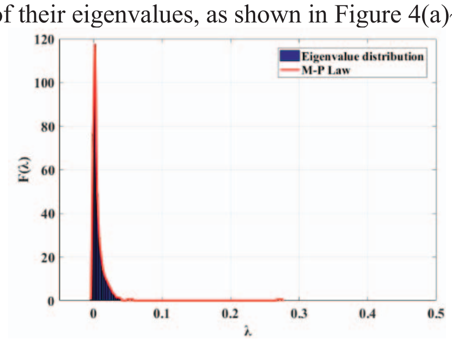


Figure 3. Original vibration signal of IMS rolling bearing 1

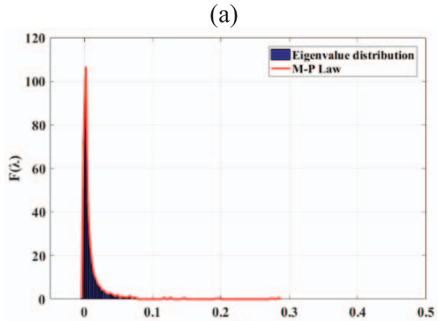
The bearing data information at different moments was selected, we repeated constructing similar matrix 9 times, and a large-scale random feature matrix $X(t_i) \in C^{400\times500}$ ($c = 0.8$) was obtained through a series of operations as shown in Section 3.1. Meanwhile, we extracted the maximum and minimum eigenvalues, and degradation index D was constructed to describe the full life history of bearing.

4.2. The Applicability of M-P Law in Bearing Dateset

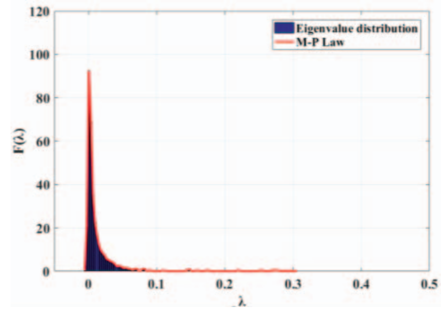
The key to using M-P law in field of bearing condition monitoring is to verify the applicability of this theory in bearing signal decomposition. The real-time collected vibration signal of rolling bearing was analyzed in this section, and bearing data from normal state, early degradation, mid-term degradation and severe degradation stages were extracted respectively. According to the method described in Section 3.1, the probability density distribution of eigenvalues after matrix decomposition was investigated, we can describe the different degradation state of bearings by studying the change of their eigenvalues, as shown in Figure 4(a)~(d).



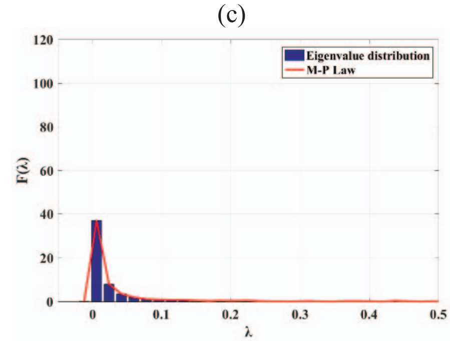
(a)



(b)



(c)



(d)

Figure 4. Probability density distribution of eigenvalues：(a) Normal state；(b) Early degradation；(c) Mid-term degradation；(d) Severe degradation.

Figure 4 (a)~(d) correspond to the probability density distribution of eigenvalues in IMS bearing life data under normal state, early degradation, middle degradation and severe degradation respectively. The vibration data collected under the normal state of the bearing has a small difference and a concentrated probability density distribution. However, when a fault occurs at a certain point in the bearing, the vibration data collected by the sensor will fluctuate, which means the bearing data collected at the same moment will vary at this stage, and the distribution difference of the data collected at the same moment will continue to increase as the fault continues to intensify. In this paper, the M-P law is used to analyze the data collected under different states of the bearing, we can achieve monitoring of bearing status by investigating the range of maximum and minimum eigenvalues. As can be seen from Figure 4 that the probability density distribution of eigenvalue fluctuates within a fixed range under normal conditions, and this distribution is relatively concentrated, which conforms to the M-P law in the random matrix theory. After matrix decomposition at this time, the maximum eigenvalue and minimum eigenvalue of the data are relatively stable, and the distribution range is relatively centralized. After the IMS rolling bearing enters degraded state, as the fault intensifies, the concentration of the eigenvalues continues to decrease, the convergence trend weakens, and the maximum and minimum eigenvalues begin to gradually change. The maximum eigenvalues show an increasing trend, while the minimum eigenvalues show a decreasing trend. Therefore, the degradation process of bearings can be studied based on the trend of changes in the maximum and minimum eigenvalues in this paper.

4.3. Early Anomaly Detection of IMS Rolling Bearing

The distribution characteristics of the eigenvalues at different stages have been verified in Section 4.2. The difference of eigenvalues index constructed is used to assess the degradation process of IMS bearing, and the trend of index changes is shown in Figure 5. When bearing 1 starts to operate in normal state, the amplitude of the index is small and the change trend is relatively stable. When fault occurs in the later stage and the fault intensifies, the amplitude of the index is large and the trend change is more significant. To verify the denoising performance of the algorithm, we conducted comparative experiments using traditional kurtosis index (Figure 6). Comparing Figures 5 and 6, we can significantly observe that the change of difference index is generally smooth, and this index has small fluctuations, which can prove the effectiveness of the index proposed in this article. At the same time, the 3 σ criterion is used to set a detection threshold for detecting early abnormal states of IMS bearing. The method described in this article detected early anomaly in IMS bearings at 5330 min, which is consistent with the results detected in [16]. Meanwhile, compared with the traditional kurtosis index (6490 min), it can detect the occurrence of bearing anomaly 19.3 hours in advance, which has great significance for timely maintenance for equipment.

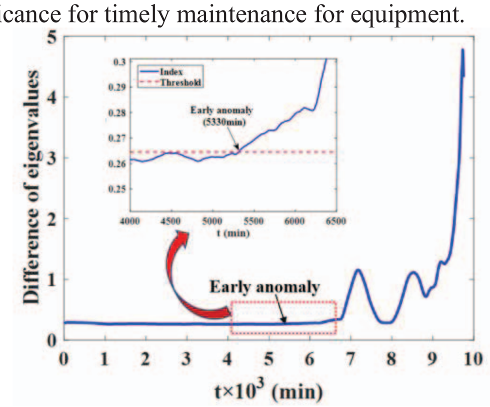


Figure 5. Early anomaly detection based on difference of eigenvalues index

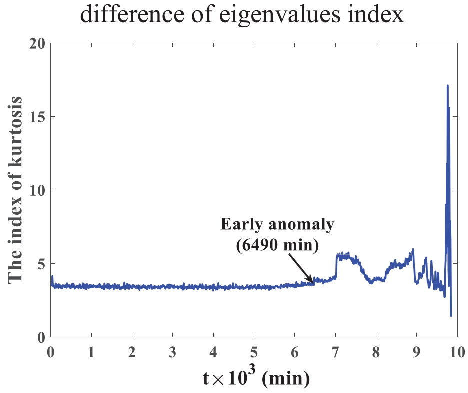


Figure 6. Early anomaly detection based on kurtosis index

4.4. Classification of different degradation stages of bearing

In order to better describe the degradation process of IMS bearing after entering abnormal condition, the vibration data of bearings after 4500 min of operation was assessed and analyzed. Dividing the bearing whole life into four different states based on the change trend of index curve, as shown in Figure 7. The bearing 1 is in normal condition before 5330 min, the index is relatively stable, and with very small amplitude changes; 5330~6280 min is defined as the early degradation state, and compared with normal state, the index has a slight increase in amplitude; 6280~8840 min is regard as the mid-term degradation state, small cracks and other defects in the outer ring of bearing continue to generate during this stage, and owing high-speed operation of rolling bearing, the rolling elements continuously smooth these defects, resulting in a 'self-healing phenomenon', the occurrence of this phenomenon is due to the appearance of cracks on the surface of bearing when it reaches fatigue state. At this time, the vibration amplitude will increase. However, when the rolling element runs at high speed to smooth out this crack, the bearing surface tends to become smooth and flat again. Thus, the vibration amplitude decreases. We defined this process as the self-healing phenomenon of the bearing, this phenomenon is discussed in detail in [17]. After 8840 minutes, bearing cannot undergo self-healing phenomenon, the index increased sharply and monotonically, the bearing 1 had undergone severe degradation, and the bearing damage continued to intensify, which will lead to the imminent failure. The results of the proposed method for classifying different degradation states of bearings are nearly consistent with in [18] and [19], which proves the effectiveness and feasibility of this degraded index.

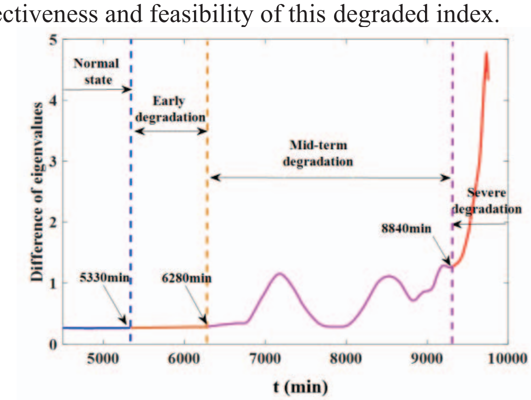


Figure 7. Classification of IMS bearing performance degradation stages

5. Conclusion

(1) The feasibility and effectiveness of M-P law in the field of rolling bearing condition detection are verified, which provides a novel method for the research in the field of rolling bearing condition monitoring base on RMT.

(2) A performance degradation assessment algorithm for rolling bearing based on M-P law is proposed, which utilizes the difference between the maximum and

minimum eigenvalues to construct detection index that can reduce the interference caused by unknown noise and improve the description ability of the index. Compared with traditional kurtosis index, it can achieve accurate detection of early abnormal states.

(3) The construction steps of index need to go through a series of operations, which are cumbersome. In the future, a method called 'sparse matrix technique' can be considered to reduce the complexity of the algorithm and improve its running speed.

(4) At present, experimental analysis has only been conducted using a matrix with a row to column ratio of 0.8. The next step is to try changing the scale of matrix to explore the accuracy and effectiveness of assessment under different ratios.

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